LOCAL AND ADAPTIVE SELECTION OF OPTIMAL PARAMETERS FOR TV REGULARIZATION MODEL AND THE FEM BASED STEREO-VISION SIMULATION

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ABSTRACT

A nonlinear variational model is proposed for the simulations of one directional disparity map using FEM(finite element method) based numerical scheme for the given Total Variation (TV) stereo problem. Our main goal is to study the appropriate selection of local smoothness parameters chosen in a uniform way and their regularization effects on the disparity image with the triangular grids as computational domain.

Keywords: ***Disparity map, regularization, Adaptive finite elements, optimization.***

1. INTRODUCTION

The efficient numerical solution of partial differential equations (PDEs) plays an important role in the engineering problems. This demand and the ever increasing computational power from current computer hardware have brought the rapid development of numerical methods for partial differential equations, a development that encompasses convergence analysis and implementation aspects of software packages and programming languages like [16 (FreeFem++)]. In this presentation we consider the TV stereo model to estimate the disparity map from two consecutive frames of same stereo scene.

The estimation of one directional disparity maps from the same image scene is one of the classical problems of image analysis research but due to the ambiguities in camera settings and aperture problem this research task is still challenging for image practitioners. Once this one directional displacement is computed accurately then it is possible to measure the distance between camera and the object. This measurement has many possible applications in driver assistance systems and auto aircrafts without pilots.

As the TV and Perona-Malik regularizations are the most popular regularization approaches for computer vision problems especially for the image restoration and Image motion problems [1-6, 8, 9, 14, 15, 18-20] and variational stereo methods [7, 10, 11, 20].

It is observed from the available literature on PDEs based approaches in image processing and computer vision [6, 9, 10, and 16] that usually the practitioners use finite difference methods using rectangular grids for the PDEs based image processing. This work provides an efficient computational approach based on adaptive finite element method (FEM) using triangular grid.

2. TV STEREO MODEL

We consider the following TV stereo model to compute the disparity map u between the stereo image pair $I:\Omega\times\mathrm{R}\rightarrow\mathrm{R}$

$$E(u)=\int_{\Omega}(\alpha\ (\psi(|\nabla u|))+D(I(x),u))\,dx....(1)$$

where

$$\psi_{\beta}(|\nabla u|)=\sqrt{\beta^{2}+|\nabla u|^{2}}\ ....(2)$$

Is the smoothness part and the

$$D(I(x),u)=(I_{x_1}+I_t)^2\ ....(3)$$

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Is called the data term which comes from the grey value constancy assumption

$$I(x+u,t+1)=I(x,t)\text{.....................................}(4)$$

We set $x=(x_1,x_2)^T \in \Omega$, here the two dimensional image domain is denoted as $\Omega \in \mathbb{R}^2$ and the terms I_{x_1}, I_t denote the derivatives with respect to x_1 and t respectively. α is strictly positive smoothness parameter.

For computation of disparity we use the energy minimization technique and we recall the following famous and basic result from the calculus of variations [6, 11]. The minimization of given two dimensional general energy functional

$$E(u)=\int_\Omega F(x_1,x_2,u,u_{x_1},u_{x_2})\,dx\text{.....................}(5)$$

Satisfy the Euler's-Lagrange Equation

$$F_u-\partial_{x_1}F_{u\,x_1}-\partial_{x_2}F_{u\,x_2}=0$$

with natural boundary condition

$$\frac{\partial u}{\partial n}=0,\ on\ \partial\Omega$$

Where n is outward normal to the boundary $\partial\Omega$, applying this direct result from the calculus of variations the computation of u is obtained from the minimization of energy functional (1) which yields the associated Euler's-Lagrange equation given as

$$0=-\mathrm{div}\left(\frac{\alpha\nabla u}{\sqrt{\beta^2+|\nabla u|^2}}\right)+(I_{x_1}{}^2u+I_{x_1}I_t)\ in\ \Omega\text{......}(6)$$

Where $\frac{\partial u}{\partial n}=0,\ on\ \partial\Omega$, β is very small smoothness parameter which is used to avoid the zero division in TV regularizer. The equation (6) is a steady state solution of the gradient system

$$\partial_t u=-\mathrm{div}\left(\frac{\alpha\nabla u}{\sqrt{\beta^2+|\nabla u|^2}}\right)+(I_{x_1}{}^2u+I_{x_1}I_t)\text{........}(7)$$

$\partial_t u$ denotes the partial derivative of u with respect to t. Variational formula for (7) can be derived as

$$(\frac{\partial u}{\partial t},v)+b(u,v)=l(f,v)\text{.................................}(8)$$

Where we set $f=-I_{x_1}I_t$, one is therefore interested to compute $u\in H^1(\Omega)$ where

$$\begin{cases} b(u,v)=\int_\Omega\left(\frac{\alpha\nabla u\cdot\nabla v}{\sqrt{\beta^2+|\nabla u|^2}}\right)dx+\int_\Omega I_{x_1}{}^2u\cdot v\ dx \\ l(u,v)=\int_\Omega f\cdot v\ dx\ \ \forall\ v\in H^1\subseteq X\text{................}(9)\end{cases}$$

H^1 is a Sobolev space which is defined as

$$H^1(\Omega)=\left\{u\in L^2(\Omega):\nabla u\in(L^2(\Omega))^2\right\}$$

For further details about Sobolev spaces and specifically $L^2(\Omega)$ spaces and their corresponding norms we refer the reader to review the basic theory of finite elements and Sobolev spaces in [12, 13].

3. FEM BASED NUMERICAL SCHEME:

We numerically solve the problem equation (8) on the following

Discrete space

$$X_h:=\left\{v_h\in C^0(\overline{\Omega})\,\middle|\,\forall K\in T_h,\ v_h\big|_K\in P_1(K)\right\}$$

Here $X_h\subset X$ is the discrete space with P_1 finite elements. C^0 is the space of continuous functions. The computational domain is considered as a triangular grid T_h with maximum size of each element $h>0$. Where $P_1(K)$ denotes the space of all polynomials functions having degree equal to one. To solve the week problem (8) on discrete space X_h the following implicit approximating scheme is proposed which is designed using both FEM (Finite Element method) and FDM (Finite difference method). The time derivative is discretized using forward difference operator.

$$(I+\tau A_\alpha)U^{K+1}=U^K+\tau L\text{...........................}(10)$$

Where

$$U=[u_1,u_2,...,u_N]\text{...}(11)$$

The $u_1,u_2,...,u_N$ are the smoothed disparity map grey values corresponding to the N number of nodes on triangular grid. The vector L is obtained from $l(f,v)$.

As the given bilinear form in equation (9) is symmetric and positive definite. The given implicit numerical scheme is unconditionally stable.

4. NUMERICAL RESULTS AND DISCUSSION

We consider a famous example of stereo pair Pentagon. The aerial view of Pentagon stereo pair with gray value images were downloaded from http:/vasc.ri.cmu.edu/idb/. Our goal is to check the disparity map for different uniformly selected values of smoothing parameter α specifically for the given FEM based numerical scheme. The value of small contrast parameter β is kept fixed for all experiments as $\beta = 0.00005$. Table1 & and Figure.1 show the Plot for the average disparity values for various choices of α. From the Figure .1 it is observed that the values of disparity map intensity slightly increases as we decrease the values of smoothness parameter α. The image results for the computed disparity maps on various fixed values of α are shown in the Figure2.(a-n). As the value of α is decreased, the grid is refined almost everywhere on the domain. We observe that some blurring effects appear in the computed disparity map with very small values of α, consequently, some useful information from the disparity map images become disappear. From overall performance of this particular numerical scheme, we have observed that with good visual quality disparity results were found when $0.5 \leq \alpha < 1$. As these variational methods have some drawback from the computational point of view in the sense that they create some computational ambiguities in the disparity map estimation at all pixels of disparity image. Keeping this all in view we propose some reliable regularization estimates and a novel regularization approach which is based on the a posteriori estimates and an intelligent algorithm which automatically identifies the damaged regions of the disparity image and regularizes. Such type of the intelligent regularization control for this problem will appear in our forth coming papers.

5. CONCLUSION

Study of locally adaptive selection of smoothness parameters is given in this paper for the disparity map estimation from successful implementation of the variational model (1) using the Finite element method on the triangular grid as domain of computation. The observed results are given in table.1, Figure.1 and the obtained disparity map images are given in Figure. 2 (a-n). From overall performance of this particular numerical scheme, we have observed that with good visual quality disparity map results were found when $0.5 \leq \alpha < 1$.

Table 1.Average disparity values for various values of α

alpha	Average Disparity
1000	0.0823566
500	0.109637
200	0.146255
100	0.17477
50	0.20208
20	0.23333
5	0.265949
0.9	0.291454
0.5	0.3006
0.1	0.348697
0.01	0.527864
0.001	0.859375

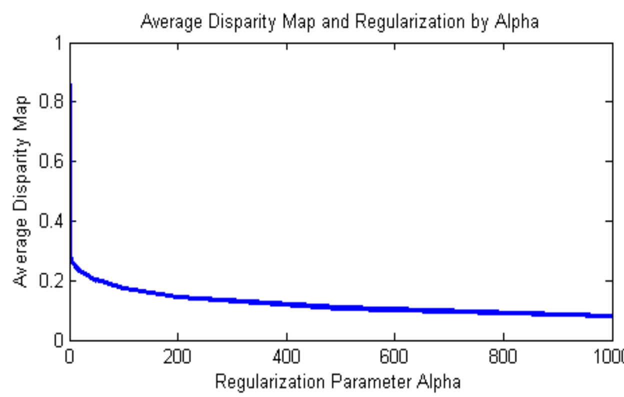
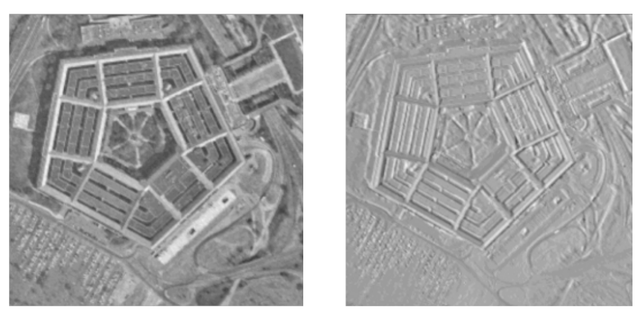
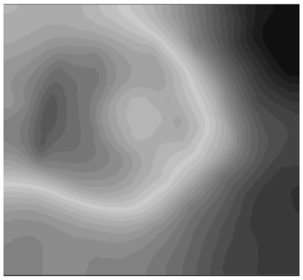


Figure 1. The plot of Average Disparity map for chosen local smoothness parameters.

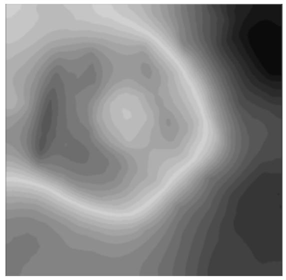


(a) Left Image Pentagon Stereo Pair

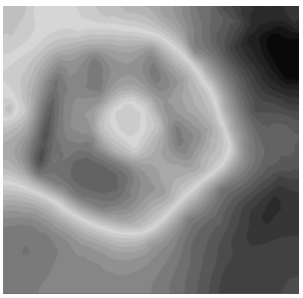
(b) Time Derivative (f_1,f_2)



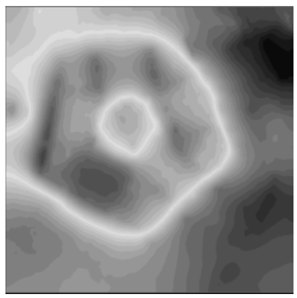
(c) Stereo Depth at $\alpha = 1000$



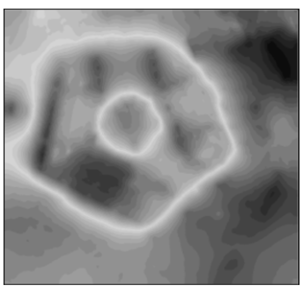
(d) Stereo Depth at $\alpha = 500$



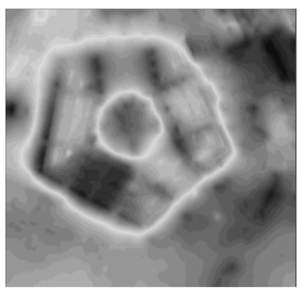
(e) Stereo Depth at $\alpha = 200$



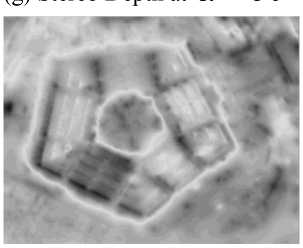
(f) Stereo Depth at $\alpha = 100$



(g) Stereo Depth at $\alpha = 50$



(h) Stereo Depth at $\alpha = 20$

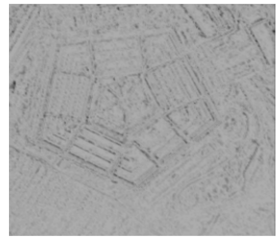


(i) Stereo Depth at $\alpha = 5$

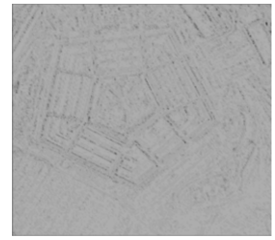
(j) Stereo Depth at $\alpha = 0.9$

(k) Stereo Depth at $\alpha = 0.5$

(l) Stereo Depth at $\alpha = 0.1$



(m) Stereo Depth at $\alpha = 0.01$



(n) Stereo Depth at $\alpha = 0.001$

Figure 2. Computed Disparity map results for various choices of the regularization parameter α .

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