FRAUNHOFER DIFFRACTION COMPUTATION OF HOLOGRAMS FOR FICTITIOUS OBJECTS

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ABSTRACT

When a hologram of non-existing object is desired one can compute it mathematically. Such type of holograms are known as computer generated holograms. In this paper such type of holography presented with an experiment of ordinary photographic recording arrangements. The objective of this work is to design computer generated hologram of a fictitious object and coding of that hologram in computer language, finally plotting of that hologram on a paper. Then reconstruction of that object is done from computer generated printed hologram by use of ordinary photographic arrangements. The fictitious objects selected for this work were horizontal lines, vertical lines and the letters comprising of my name FATTAH the coding of this program has been done in MATLAB.

Kev Words: Holographv. Diffraction. Optical Fourier Transform. CGH

1. INTRODUCTION

Images and photographs are since long in use for identification and location of objects. Photography was introduced after the invention of optical lens. Photograph provides in-depth study of an area; indicates the presence of objects of interest in that area and their location. Images from satellite depict weather changes, agriculture growth, geological and global actions. Special images may contain additional scientific information including object spectral characteristics, velocity, temperature, etc. Hologram is a record of the interaction of two beams of coherent light which are mutually correlated in the form of microscopic pattern of interference fringes. One beam is reference and other one is object beam, the process of fabrication of hologram is known as holography. An ordinary hologram is produced by recording the interference pattern, which is caused by an object wave and a reference wave On this principle Dennis Gabor became the first to record the object wave and name it as hologram in 1948 [1]. But when a hologram is desired of an object, which does not exist physically but is known in mathematical terms, one can compute the hologram. The simulated hologram is drawn by high resolution plotting machine and subsequently photographically reduced in size. This type of hologram is known as computer generated hologram [2,3]. These holograms are Fourier transform holograms. Main advantage of computer generated holography is the fact that the object does not required to exist.

In 1967 another milestone was achieved by introducing a new concept of holography that was "computer generated holography" by A W Lohman and D. P. Paris [2]. Recent advances in three dimensional display technologies have shifted attention to the possibility of true holographic displays [4].

The objectives of this work is construction of computer generated hologram. The class of holography that have been selected for computer simulation is Fourier Transform Holography. The objects used for computer generated holography are horizontal line, vertical line, and the word **FATTAH**. These objects are made of rectangular apertures.

2. DIFFRACTION THEORY

When a light is obstructed by solid body it will create complex shadow of bright and dark regions and can be viewed by placing and screen on other side of body. The phenomenon is known as diffraction. The effect is general characteristic of wave phenomena occurring whenever a portion of wavefront is obstructed in some way when it is propagating in rectilinear direction. If a region of wavefront is altered in amplitude or phase whether it is done by opaque or transparent obstacle diffraction will occur. The various segments of the wavefront that propagate beyond the obstacle interfere, causing the particular energy density referred to as diffraction pattern [5, 6]. The figure 1 shows the diffraction patterns where

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structured fringes represent the density of diffracted waves.

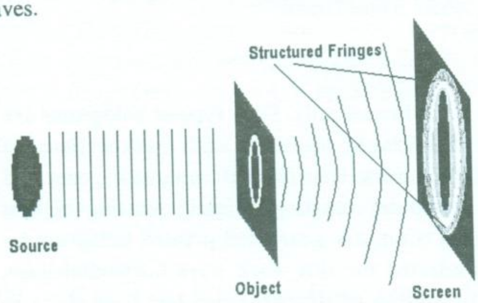

Figure 1: General description of diffraction

3. FRAUNHOFER AND FRESNEL DIFFRACTIONS.

Consider aperture shaped object smaller in size made in solid shield is illuminated by plane waves originated from a point source located at distance ***d***. If image screen is placed at reasonable distance other side of object (i-e aperture sheet) then image of aperture can be observed onto the screen along with some light fringes around the boundaries of aperture. If the image screen is moved farther away from aperture, the image of aperture becomes structured and the fringes become more prominent. This is known as near field or Fresnel diffraction. If the image screen moved farther from the object one stage will come where there will be no change occurs in fringes or image of object only size of image will be changed. Such phenomenon is known as Fraunhofer or far field diffraction.

4. COMPUTER GENERATED HOLOGRAPHY

Hologram is the recording of interference pattern that caused by object beams and a reference beam. Under usual circumstances, the hologram fringes may locally be viewed as arcs of circles, whose radii of curvature are large compared with the local fringe period [7]. Hologram can be desired those objects which does not exist physically but can be deemed in mathematical terms. Such type of holograms is known as computer generated holograms [8]. It is important to mention that computer generated hologram does not need any optical system but can be computed mathematically by using digital computers [9]. Computer generated holograms presented here are binary. In this research creation of hologram is done by computer simulation of Fourier Transform Holography based on Fraunhofer diffraction.

5. DESIGN STEPS COMPUTER GENERATED HOLOGRAPHY

To design computer simulated hologram following steps are adopted.

i. Object specification
ii. Calculation of amplitude and phase of diffracted light from the object.
iii. Sampling of the Fourier Transform
iv. Computation of hologram by calculation of interference between reference beam and object beam
v. Coding Schemes
vi. Plotting of Hologram.

Here in this research objects are considered as assembly of rectangular apertures.

5.1 CALCULATION OF FRAUNHOFER DIFFRACTION PATTERN FOR RECTANGULAR APERTURE

Consider configuration of Rectangular Aperture as shown in Figure 2. Amplitude and phase of light incident at a location X,Y on an image screen located at a distance Z from aperture. The width of aperture is w and height of aperture is h where

$w = x_1 - x_2$ and $h = y_1 - y_2$

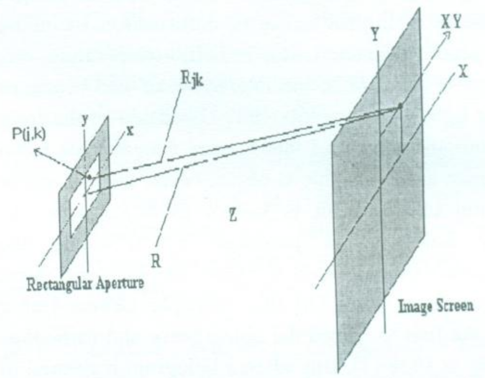

Figure 2: Light distribution by rectangular aperture to an image screen

Consider a point location P(jk) in aperture then R_{jk} the distance between point jk on aperture and X,Y on image screen will be

$$R_{jk} = \sqrt{(X - x_i)^2 + (Y - y_k)^2 + Z^2} \qquad (4\text{-}1)$$

let $R = \sqrt{X^2 + Y^2 + Z^2}$

Then, $R_{jk} = R\sqrt{1 - \frac{2Xx_j}{R^2} + \frac{x_j^2}{R^2} - \frac{2Yy_k}{R^2} + \frac{y_k^2}{R^2}}$

as $R>>x_j$, y_k therefore we can neglect the terms x_j^2/R^2 and y_k^2/R^2 therefore

$$R_{jk} = R - \frac{Xx_j}{R} - \frac{Yy_k}{R}$$

Now if we consider the diffraction from whole aperture (X,Y) is E(X,Y) and $R_{jk} = R'$ then we can write

$$E(X,Y) = \int_{y_2}^{y_1}\int_{x_2}^{x_1} ae^{i\frac{2\pi R'}{\lambda}} dxdy \qquad (4\text{-}2)$$

substituting the value of R' in Equation (4-2) then

$$E(X,Y) = \int_{y_2}^{y_1}\int_{x_2}^{x_1} ae^{i\frac{2\pi}{\lambda}\left(R - \frac{Xx}{R} - \frac{Yy}{R}\right)} dxdy$$

Or

$$E(X,Y) = ae^{i\frac{2\pi R}{\lambda}}(x_1 - x_2)e^{-i\frac{\pi X(x_1+x_2)}{\lambda R}} * \text{sinc}\left(\frac{\pi X(x_1 - x_2)}{\lambda R}\right) * (y_1 - y_2)e^{-i\frac{\pi Y(y_1+y_2)}{\lambda R}} \text{sinc}\left(\frac{\pi . Y(y_1 - y_2)}{\lambda R}\right) \qquad (4\text{-}3)$$

Equation (4-3) is the generalized equation for Fraunhofer diffraction by a rectangular aperture on an image screen located at a distance Z from the aperture.

5.2 INSERTION OF LENS FUNCTION

Fraunhofer diffraction can be achieved when image screen is at infinite distance so theoretically value of Z is infinite to avoid this problem we place a lens between aperture and image screen so that same phenomenon can be achieved by placing image screen at one f distance from lens. By considering Equation (4-1) and solving R_{jk} in the form of parallel distance Z, gives:

$$R_{jk} = \sqrt{X^2 - 2Xx_j + x_j^2 + Y^2 - 2Yy_k + y_k^2 + Z^2}$$
$$= Z\left[1 + \frac{X^2 + Y^2}{2Z^2} + \frac{x_j^2 + y_k^2}{2Z^2} - \frac{x_j X}{Z^2} - \frac{y_k Y}{Z^2} ..\right]$$

if we ignore all terms of high powers of xj and y_k which are divided by high power terms of Z and replace 'x_j' by 'x', 'y_k' by .'y' then

$$R_{jk} \cong Z + \frac{X^2 + Y^2}{2Z} - \frac{xX + yY}{Z} \qquad (4\text{-}4)$$

now take $R_{jk}=R'$ and substituting 4-4 in Equation (4-2) that is

$$E(X,Y) = \int_{y_2}^{y_1}\int_{x_2}^{x_1} ae^{i\frac{2\pi}{\lambda}\left(Z + \frac{X^2+Y^2}{2Z} - \frac{xX+yY}{Z}\right)} dxdy$$

or

$$E(X,Y) = ae^{i\frac{2\pi}{\lambda}Z\left[1+\frac{X^2+Y^2}{2Z^2}\right]}\int_{y_2}^{y_1}\int_{x_2}^{x_1} e^{i\frac{2\pi}{\lambda}\left[\frac{xX+yY}{Z}\right]} dxdy$$

where as Z>>X, Z>>Y in other words $1>>(X^2+Y^2)/Z^2$ therefore we can write

$$E(X,Y) = ae^{i\frac{2\pi Z}{\lambda}}\int_{y_2}^{y_1}\int_{x_2}^{x_1} e^{-i\frac{2\pi}{\lambda}\left[\frac{xX+yY}{Z}\right]} dxdy \qquad (4\text{-}5)$$

now if we analyse Equation (4-5) we will see that all terms on left hand side of integral signs are constants and will perform a uniform action on all diffracting light waves in image screen so we can eliminate them by one suffix C.

$$E(X,Y) = C\int_{y_2}^{y_1}\int_{x_2}^{x_1} e^{-i\frac{2\pi}{\lambda}\left[\frac{xX+yY}{Z}\right]} dxdy \qquad (4\text{-}6)$$

now if we place a well corrected lens of focal length *f* away from the aperture and image screen is placed at rear focal plane then same phenomenon (ie far field diffraction) will be achieved as shown in Figure 3.

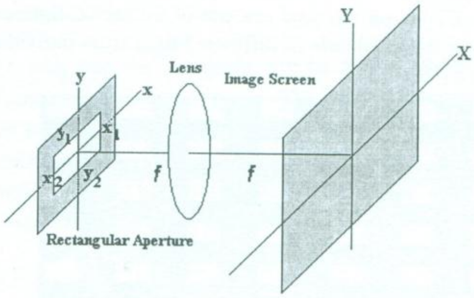

Figure 3: Use of lens in holography

Then Equation (4-6) becomes

$$E(X,Y) = C\int_{y_2}^{y_1}\int_{x_2}^{x_1} e^{-i\frac{2\pi}{\lambda}\left[\frac{xX+yY}{f}\right]} dxdy$$
$$= C(x_1 - x_2)(y_1 - y_2)e^{-i\frac{\pi(x_1+x_2)X}{f\lambda}} e^{-i\frac{\pi(y_1+y_2)Y}{f\lambda}} * \sin c\left(\frac{\pi(x_1 - x_2)X}{f\lambda}\right) * \sin c\left(\frac{\pi(y_1 - y_2)Y}{f\lambda}\right)$$

whereas x1-x2=width 'w' of aperture and y1-y2=height 'h' of aperture therefore

$$E(X,Y) = Cwhe^{-i\frac{\pi(x_1+x_2)X}{f\lambda}} e^{-i\frac{\pi(y_1+y_2)Y}{f\lambda}} * \sin c\left(\frac{\pi(x_1-x_2)X}{f\lambda}\right) * \sin c\left(\frac{\pi(y_1-y_2)Y}{f\lambda}\right) \quad (4\text{-}7)$$

Equation (4-7) is the generalized equation for light diffraction from an object of rectangular aperture shaped on an image screen at single point whose location is X,Y in image screen. Whereas f is the focal length of lens which is placed between image screen and object as shown in Figure 3. Equation (4-7) shows the complex amplitude of diffracted light from single rectangle location X,Y in Fourier plane (i-e image screen). Now consider the situation where an object has more then one apertures then the resultant complex amplitude of diffracted light on single point (X,Y) will be the sum of all complex amplitudes of light diffracting from each aperture individual.

Consider Figure 4 where an object comprises of $m \times n$ number of rectangle apertures then total light contribution on location XY will be

$$E_T(X,Y) = \sum_m \sum_n E_I(X,Y)$$

Where ET shows the total amount of diffracted light and EI shows the amplitude of diffracted light from individual aperture [10].

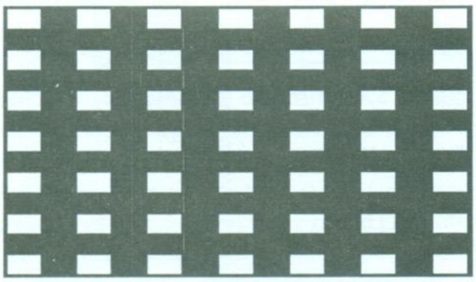

Figure 4: Object consists of m∗n number of transparent apertures.

5.3 COMPUTATION OF HOLOGRAM

Hologram is record of an interference pattern formed by superposition of waves, diffracted from an object and the reference wave. A general setting of an ordinary holography is shown in Figure 5 where object is a rectangular aperture. The object wave O is diffracted wave coming from object incident on the image screen and interfere with reference wave R. Then the distribution of intensity recorded on the image screen is given as

$$H = |R+O|^2.$$
$$H = |R|^2 + |O|^2 + RO^* + OR^* \quad (4\text{-}8)$$

Where

- R = reference wave
- O = object wave
- R^* = conjugate of R
- O^* = conjugate of O

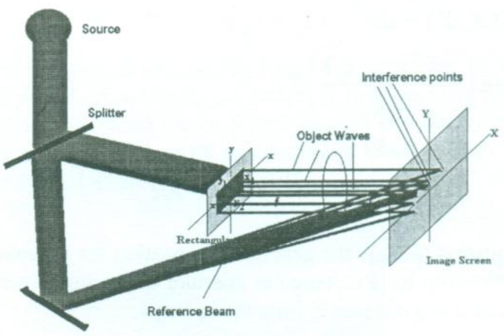

Figure 5: General arrangements for holography

In Equation (4-8) first two terms corresponds the intensities of reference beam and object beams whereas third and fourth terms represents the interference pattern between reference wave and object wave, these are the terms which contains phase and amplitude information of object and reference beam. Our computer program will calculate only third and fourth terms as given below

$$H(X,Y) = RO^* + OR^*$$
$$= 2OR\cos(\varphi_1 - \varphi_2)$$

where φ1 is the phase of object beam and φ2 is the phase of reference beam. In computation of computer simulated hologram the computation of diffracted light distribution on image screen is the object beam. Whereas for reference beam we introduce mathematically a beam of same wavelength as used in calculation of object or diffracted beam.

6. EXPERIMENTS AND RESULTS

For designing and coding of computer generated holograms various mathematically defined objects are used. These objects are Horizontal Line, Vertical Line, Plus Sign, and word FATTAH. These objects are composition of rectangular apertures.

6.1 COMPUTER GENERATED HOLOGRAM OF HORIZONTAL LINE

We take the object as horizontal line in the form of rectangular aperture as shown in Figure 6. To compute the hologram of this object Equation (4-7) is used i.e.

$$E(X,Y) = Cwhe^{-i\frac{\pi(x_1+x_2)X}{f\lambda}} e^{-i\frac{\pi(y_1+y_2)Y}{f\lambda}} * \sin c\left(\frac{\pi(x_1-x_2)X}{f\lambda}\right) * \sin c\left(\frac{\pi(y_1-y_2)Y}{f\lambda}\right)$$

The values of variables are

- **x1 = 1 millimeter**
- **x 2 = -1 mm**
- **y1 = 0.1 mm**
- **y2 = -0.1 mm**
- **f = 4000 mm**
- **$\lambda = 6.2*10^{-4}$ mm**

X,Y are coordinates of Fourier plane where hologram will be computed, so we will select discrete values of X,Y. These discrete values will be in accordance with sampling theorem. For horizontal line hologram two dimensional matrix of 41*41 order will be used to store the phases and amplitudes of hologram with respect to the discrete values of X and Y coordinates. Where X starts from -200 to +200 millimeters with step size of 10 millimeters and Y starts from -200 to +200 millimeters with step size of 10 millimeters in other words sampling rate is 10 millimeters.

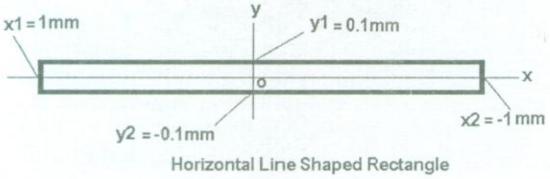

Figure 6: Horizontal line shaped rectangular Aperture

For reference beam we use a plane beam (of same wavelength as that of object beam) of unity magnitude, tilted at 45 degrees with X-Z plane in Fourier plane.

6.2 COMPUTER PROGRAMMING FOR HOLOGRAM.

Computer program used for computation of computer generated hologram is written in MATLAB software. The computer program is divided into five steps:

In first step it will calculate the amplitude and phase of diffracted light wave from object that is Fourier transform of object and store it into a two dimensional array form.
In second step it will calculate the interference pattern between reference plane wave and object wave and store them in a two dimensional array form.

In third step enhancement of amplitude of hologram will be done. The reason for enhancement is such that the amplitudes of higher orders of Fourier transform of object are so small that they are suppressed by the values of amplitudes around the origin of aperture. This difference can be observed by comparing the holograms made with amplitude enhancement shown in Figure 7 and without amplitude enhancement shown in Figures 8. Figure 9 and figure 10 three dimensional plots (surf-plot) with and without amplitude enhancement of holograms.

In fourth step computer program will scale the amplitude of hologram up-to certain limit so that all values below that limit will not be plotted. This is because to have some opaque contrast between two prominent orders in reconstruction

In fifth step the hologram will be graphically plotted on monitor or printed on paper. The graphical representation of hologram is in the form of rectangular opaque dots in such a way that height of dot represents the amplitude of hologram and horizontal offset shift is proportional to the phase of hologram at that discrete location.

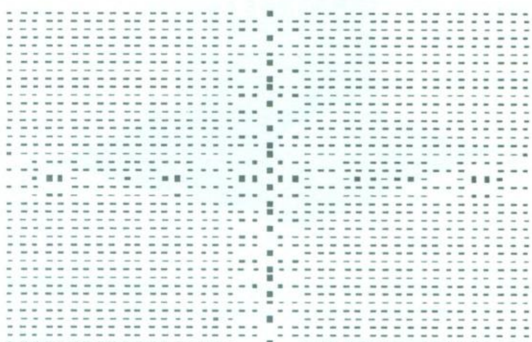

Figure 7: Computer Generated Hologram of Horizontal Line with amplitude enhancement

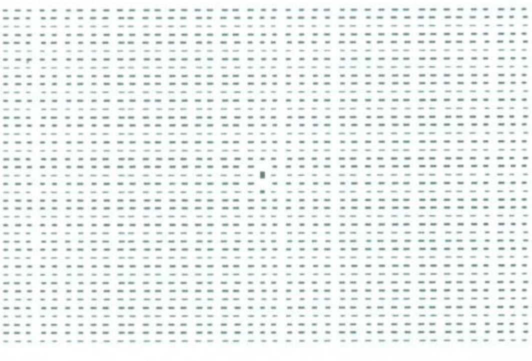

Figure 8: Computer Generated Hologram of Horizontal Line without amplitude enhancement

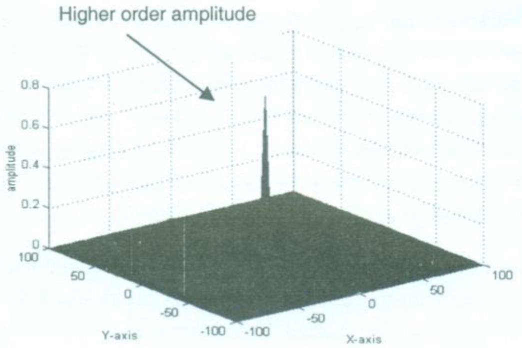

Figure 9: Surf plot of amplitude of computed hologram of horizontal line without amplitude enhancement

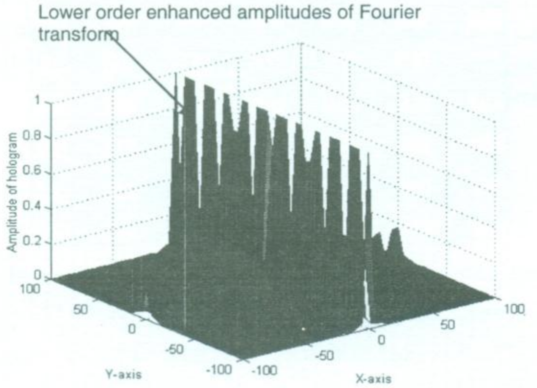

Figure 10: Surf plot of amplitude of computed hologram of horizontal line after amplitude enhancement.

7 CONCLUSION

In this paper computer simulation of Fourier transform holograms, of various objects are designed and constructed. MATLAB has been used for computer programming. Good quality holograms are computed using larger arrays of the data points. The objects that are simulated for computer generated holograms are rectangular apertures, horizontal lines, vertical lines, plus sign and the word FATTAH. Use of rectangular apertures for creation of hologram can also be used to develop 3-D display system for view of 3-D objects [11]. In this work construction of computer generated hologram has been tried on photosensitive film. For this purpose we have used ordinary photographic films and camera. Therefore the recorded holograms were not of good quality. To record good quality high resolution holograms, proper holographic film and the fabrication system is suggested.

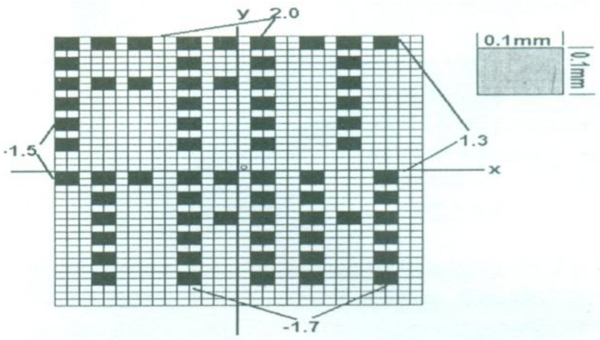

Figure 11: Object 'FATTAH

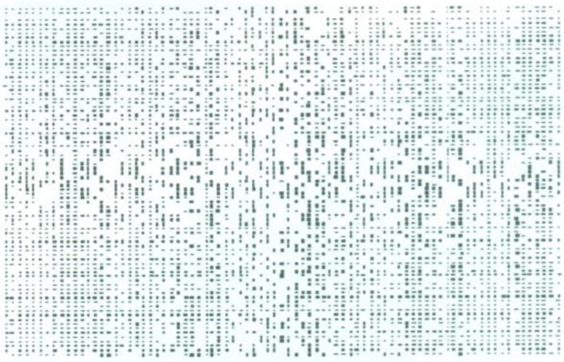

Figure 12: Computer Generated Hologram of object FATTAH

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